

**OSNOVI ELEKTRONIKE**  
**Modul elektroenergetika (2OEP3O03) (3OEP3A01)**

**1. Zadatak**

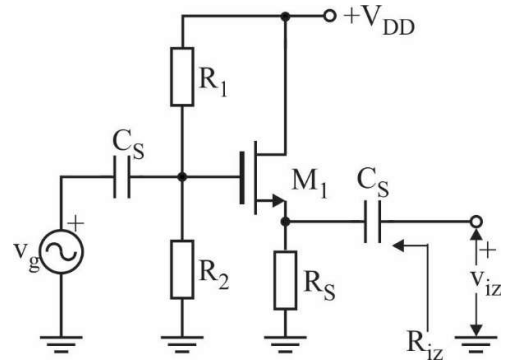
Za kolo pojačavača prikazano na slici odrediti:

a) Dinamičke parametre tranzistora: strminu  $g_m$ , izlaznu otpornost  $r_o$  i koeficijent naponskog pojačanja  $\mu$ ;

b) Naponsko pojačanje  $A_n = v_{iz}/v_g$ ;

c) Izlaznu otpornost tranzistora  $R_{iz}$ .

Elementi kola su:  $R_S = 6 \text{ k}\Omega$ ,  $R_1 = 1 \text{ M}\Omega$ ,  $R_2 = 2 \text{ M}\Omega$ ,  $V_{DD} = 12 \text{ V}$ ,  $C_S \rightarrow \infty$ . Parametri tranzistora su:  $A = 1 \text{ mA/V}^2$ ,  $V_t = 1 \text{ V}$ ,  $\lambda = 0,01 \text{ V}^{-1}$  ( $V_A = 100 \text{ V}$ ).



**Rešenje:**

a)

$$V_G = \frac{R_2}{R_1 + R_2} \cdot V_{DD}$$

$$V_S = R_S \cdot I_D$$

$$I_D = A \cdot (V_{GS} - V_t)^2$$

$$V_x = V_{GS} - V_t$$

$$V_{GS} = \frac{R_2}{R_1 + R_2} \cdot V_{DD} - R_S \cdot A \cdot (V_{GS} - V_t)^2$$

$$R_S \cdot A \cdot (V_{GS} - V_t)^2 + (V_{GS} - V_t) + V_t - \frac{R_2}{R_1 + R_2} \cdot V_{DD} = 0$$

$$R_S \cdot A \cdot V_x^2 + V_x + V_t - \frac{R_2}{R_1 + R_2} \cdot V_{DD} = 0$$

$$V_x^2 + V_x - 2 = 0$$

$$V_x = \frac{-1 \pm \sqrt{1 + 8}}{2}$$

$$V_{x1} = 1 \text{ V}$$

$$V_{x2} = -2 \text{ V}$$

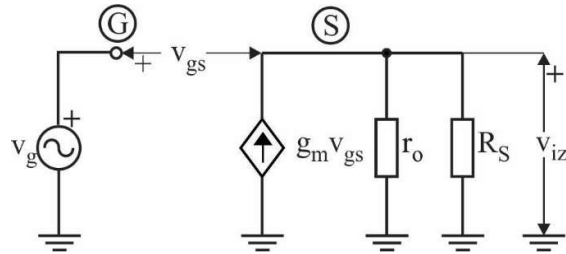
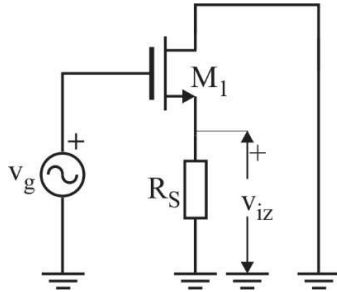
$$I_D = A \cdot V_x^2 = 1 \text{ mA}$$

$$g_m = 2 \cdot \sqrt{A \cdot I_D} = 2 \text{ mS}$$

$$r_o = \frac{1}{\lambda \cdot I_D} = 100 \text{ k}\Omega$$

$$\mu = g_m \cdot r_o = 200$$

b)



$$-g_m \cdot v_{gs} + \frac{v_s}{r_o} + \frac{v_s}{R_S} = 0$$

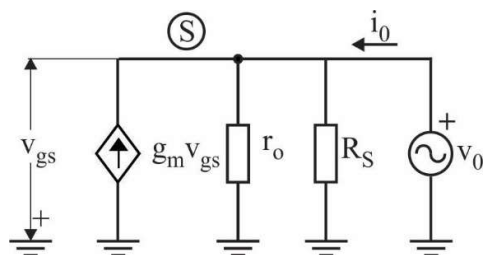
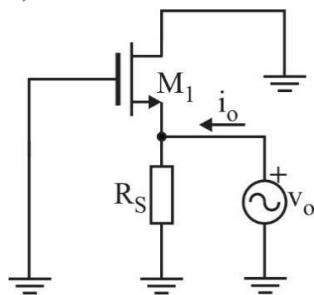
$$v_{gs} = v_g - v_s$$

$$-g_m \cdot v_g + g_m \cdot v_s + \frac{v_s}{r_o} + \frac{v_s}{R_S} = 0$$

$$v_{iz} = v_s = \frac{g_m \cdot v_g}{g_m + \frac{1}{r_o} + \frac{1}{R_S}}$$

$$A_n = \frac{v_{iz}}{v_g} = \frac{g_m}{g_m + \frac{1}{r_o} + \frac{1}{R_S}} = 0.92$$

c)



$$i_o = \frac{v_o}{R_S} + \frac{v_o}{r_o} - g_m \cdot v_{gs}$$

$$v_{gs} = -v_s$$

$$R_{iz} = \frac{v_o}{i_o} = \frac{1}{\frac{1}{R_s} + \frac{1}{r_o} + g_m} = R_s \parallel r_o \parallel \left(\frac{1}{g_m}\right) = 497 \Omega$$

$$\left(\frac{1}{g_m}\right) \ll R_s, r_o$$

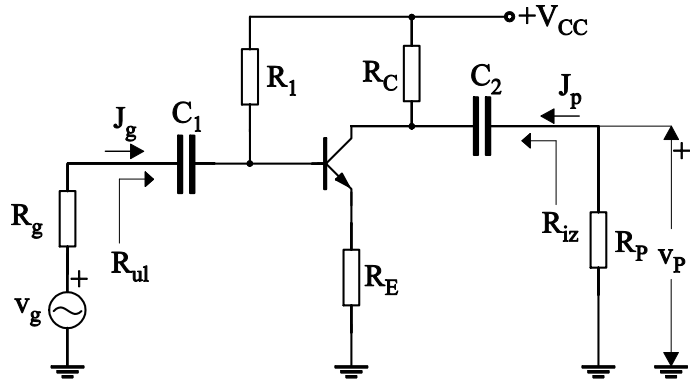
$$R_{iz} = R_s \parallel r_o \parallel \left(\frac{1}{g_m}\right) \approx g_m$$

## 2. Zadatak

Za kolo pojačavača prikazano na slici odrediti:

- Jednosmerni napon između kolektora i emitora tranzistora  $V_{CE}$ ;
- Dinamičke parametre parametre:  
 $h_{11E} = r_\pi$  i transkonduktansu  $g_m$ ;
- Naponsko pojačanje  $A_n = \frac{v_p}{v_g}$ .

Parametri tranzistora su:  $V_{BE}=0,6 \text{ V}$ ;  $h_{12E} = 0$ ;  $\beta = h_{21E} = 50$ ;  $h_{22E} = 0 \text{ S}$  ( $r_o \rightarrow \infty$ ). Elementi kola su:  $R_p = R_C = 5 \text{ k}\Omega$ ;  $R_g = 1 \text{ k}\Omega$ ;  $R_1 = 500 \text{ k}\Omega$ ;  $R_E = 200 \Omega$ ;  $V_{CC}=12 \text{ V}$ ;  $C_1 \rightarrow \infty$ ;  $C_2 \rightarrow \infty$ .  
 Temperaturski potencijal iznosi  $V_T = 26 \text{ mV}$ .



## Rešenje:

a)

$$V_{CC} - R_1 \cdot I_B - V_{BE} - R_E \cdot (1 + \beta) \cdot I_B = 0$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_1 + R_E \cdot (1 + \beta)} = 20 \mu A$$

$$V_C = V_{CC} - R_C \cdot I_C$$

$$V_E = R_E \cdot I_E = R_E \cdot (1 + \beta) \cdot I_B$$

$$V_{CE} = V_{CC} - R_C \cdot \beta \cdot I_B - R_E \cdot (1 + \beta) \cdot I_B = 6,8 \text{ V}$$

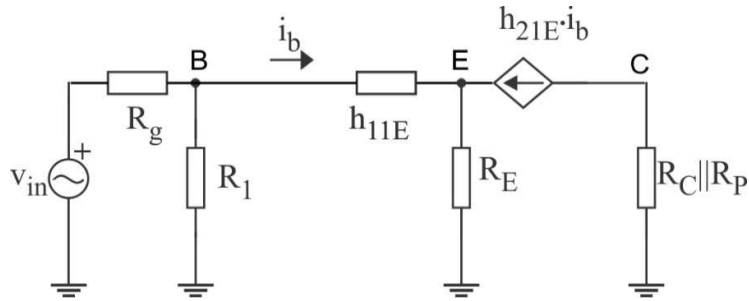
b)

$$h_{11E} = r_\pi = \frac{V_T}{I_B} = \frac{26 \text{ mV}}{20 \mu A} = 1,3 \text{ k}\Omega$$

$$h_{21E} = \beta$$

$$g_m = \frac{h_{21}}{r_\pi} = \frac{I_C}{V_T}$$

c)



$$(E) \quad \frac{v_e}{R_E} - i_b - h_{21E} \cdot i_b = 0$$

$$(B) \quad \frac{v_b}{R_1} + \frac{v_b - v_g}{R_g} + i_b = 0$$

$$(C) \quad \frac{v_c}{R_C \parallel R_P} - h_{21E} \cdot i_b = 0$$

$$i_b = \frac{v_b - v_e}{h_{11E}}$$

$$(E) \quad v_e = R_E \cdot i_b \cdot (1 + h_{21E})$$

$$(B) \quad v_b = v_g \cdot \frac{R_1}{R_1 + R_g} - i_b \cdot \frac{R_1 \cdot R_g}{R_1 + R_g}$$

$$i_b = \frac{v_g \cdot \frac{R_1}{R_1 + R_g}}{R_E \cdot (1 + h_{21E}) + \frac{R_1 \cdot R_g}{R_1 + R_g} + h_{11E}}$$

$$v_p = v_c = -h_{21E} \cdot i_b \cdot R_C \parallel R_P$$

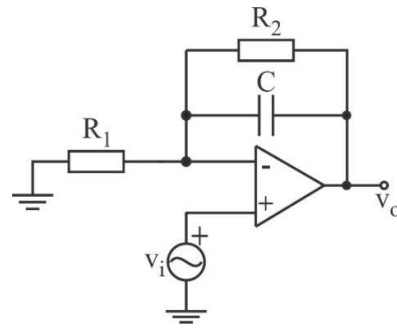
$$A_n = \frac{v_c}{v_p} = - \frac{h_{21E} \cdot R_C \parallel R_P \cdot \frac{R_1}{R_1 + R_g}}{R_E \cdot (1 + h_{21E}) + \frac{R_1 \cdot R_g}{R_1 + R_g} + h_{11E}} = 10$$

### 3. Zadatak

Za kolo sa slike odrediti:

- Prenosnu funkciju  $T(s) = \frac{V_o(s)}{V_i(s)}$  ;
- Jednosmerno pojačanje;
- Graničnu frekvenciju.

Poznato je:  $R_1 = R_2 = R = 10 \text{ k}\Omega$ ;  $C = 20 \text{ nF}$ .



Rešenje:

$$\frac{v_1}{R_1} + \frac{v_1 - v_o}{R_2} + s \cdot C \cdot (v_1 - v_o) = 0$$
$$v_1 = v_i$$

$$T(s) = \frac{v_o}{v_i} = \frac{R_1 + R_2 + s \cdot C \cdot R_1 \cdot R_2}{R_1 + s \cdot C \cdot R_1 \cdot R_2}$$
$$T(s) = \frac{R_1 + R_2}{R_1} \cdot \frac{1 + s \cdot C \cdot \frac{R_1 \cdot R_2}{R_1 + R_2}}{1 + s \cdot C \cdot R_2}$$

Ovaj filter je **propusnik niskih frekvencija** jer kada frekvencija teži nuli pojačanje teži konačnoj vrednosti, a kada frekvencija teži beskonačnosti pojačanje teži nuli. Za propusnik niskih frekvencija nominalno pojačanje,  $T_o$ , se dobija za  $s=0$ .

$$T(s) = T_o \cdot \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}}$$

Nominalno pojačanje je i jednosmerno pojačanje  $T_o = \frac{R_2 + R_1}{R_1} = 2$

$$\text{Frekvencija pola } \omega_p = \frac{1}{C \cdot R_2} = 0,5 \cdot 10^4 \frac{\text{rad}}{\text{s}}$$

$$\text{Frekvencija nule } \omega_z = \frac{1}{C \cdot R_1 \parallel R_2} = 1 \cdot 10^4 \frac{\text{rad}}{\text{s}}$$

Amplitudska karakteristika se dobija kao moduo prenosne funkcije kola:

$$|T(j\omega)| = \left| T_o \cdot \frac{1 + \frac{j \cdot \omega}{\omega_z}}{1 + \frac{j \cdot \omega}{\omega_p}} \right| = |T_o| \cdot \sqrt{\frac{1 + \left(\frac{\omega}{\omega_z}\right)^2}{1 + \left(\frac{\omega}{\omega_p}\right)^2}}$$

Granična frekvencija,  $\omega_{3dB}$ , je frekvencija na kojoj je moduo pojačanje manji  $\frac{1}{\sqrt{2}}$ puta u odnosu na nominalno pojačanje.

$$|T(j\omega_{3dB})| = |T_o| \cdot \frac{1}{\sqrt{2}}$$

$$\frac{1 + \left(\frac{\omega_{3dB}}{\omega_z}\right)^2}{1 + \left(\frac{\omega_{3dB}}{\omega_p}\right)^2} = \frac{1}{2}$$

$$\omega_{3dB} = \frac{1}{\sqrt{\frac{1}{\omega_p^2} - \frac{2}{\omega_z^2}}} = \sqrt{2} \cdot \omega_p = 0,71 \cdot 10^4 \frac{rad}{s}$$

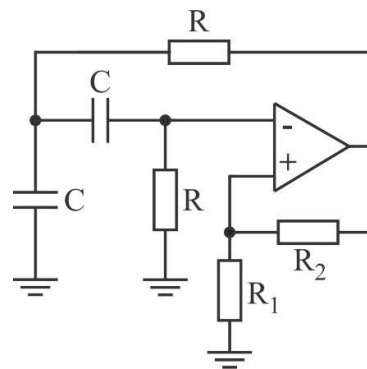
#### 4. Zadatak

U kolu oscilatora prikazanog na slici poznato je:

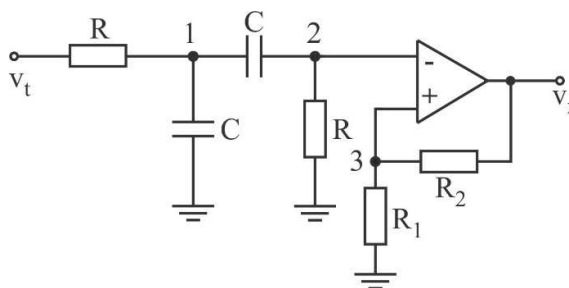
$$R = 1 \text{ k}\Omega \quad C = 10 \text{ nF} \quad R_1 = 1 \text{ k}\Omega.$$

Operacioni pojačavač je idealan. Odrediti:

- Kružno pojačanje
- Frekvenciju oscilacija
- Vrednost otpornika  $R_2$  pri kojoj nastaju oscilacije.



Rešenje:



$$(V_1 - V_t) \cdot \frac{1}{R} + s \cdot C \cdot V_1 + (V_1 - V_2) \cdot sC = 0$$

$$V_2 \cdot \frac{1}{R} + (V_2 - V_1) \cdot s \cdot C = 0$$

$$V_3 \cdot \frac{1}{R_1} + (V_3 - V_x) \cdot \frac{1}{R_2} = 0$$

$$V_2 = V_3$$

$$A \cdot B = \frac{V_x}{V_t} = \frac{R_1 + R_2}{R_1} \cdot \frac{s \cdot C \cdot R}{1 + 3 \cdot s \cdot C \cdot R + (s \cdot C \cdot R)^2}$$

$$A \cdot B(j\omega) = \frac{R_1 + R_2}{R_1} \cdot \frac{j \cdot \omega \cdot C \cdot R}{1 + 3 \cdot j \cdot \omega \cdot C \cdot R - \omega^2 \cdot (C \cdot R)^2}$$

Prema Barkhauzenovom uslovu oscilovanja neophodan uslov da nastupe oscilacije je da kružno pojačanje iznosi 1.

$$A \cdot B(j\omega) = 1$$

$$\frac{R_1 + R_2}{R_1} \cdot \frac{j \cdot \omega \cdot C \cdot R}{1 + 3 \cdot j \cdot \omega \cdot C \cdot R - \omega^2 \cdot (C \cdot R)^2} = 1$$

$$1 + 3 \cdot j \cdot \omega \cdot C \cdot R - \omega^2 (C \cdot R)^2 - \left( \frac{R_1 + R_2}{R_1} \right) \cdot j \cdot \omega \cdot C \cdot R = 0$$

Da bi ova jednačina bila ispunjena neophodno je da zbir realnih sabiraka bude jednak nuli i da zbir imaginarnih sabiraka bude jednak nuli. Odavde se dobijaju dve jednačine. Iz jedne od njih dobijamo uslov oscilovanja a iz druge frekvenciju oscilovanja.

$$1 - \omega^2 (C \cdot R)^2 = 0$$

$$3 \cdot j \cdot \omega \cdot C \cdot R - \left( \frac{R_1 + R_2}{R_1} \right) \cdot j \cdot \omega \cdot C \cdot R = 0$$

$$\omega_o = \frac{1}{R \cdot C}$$

$$\frac{R_2}{R_1} = 2$$

Frekvencija oscilovanja

Uslov oscilovanja

$$\omega_o = 10^5 \frac{\text{rad}}{\text{s}}$$

$$R_2 = 2 \text{ k}\Omega$$