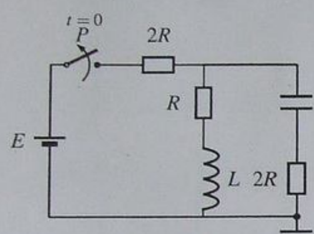
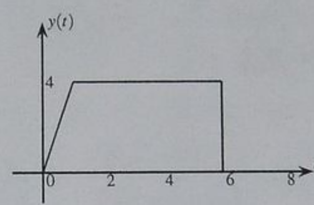
ELEKTRONSKI FAKULTET U NIŠU
KATEDRA ZA ELEKTRONIKU **mart 2012.god**
PREDMET: SIGNALI I SISTEMI

Zadaci:

1. U kolu sa slike 1 u trenutku $t = 0$ prekidač se otvara.
a) Odrediti početne uslove i nacrtati kolo u frekevencijskom domenu za $t > 0^+$ ako je $E = 6V$, $R = 2\Omega$, $C = 2F$, $L = 3H$.
b) Napisati sistem jednačina za kolo u frekvencijskom domenu i izraziti napon na kondenzatoru.



Slika 1:



Slika 2:

c) Signal sa slike 2 izraziti preko Heaviside-ovih funkcija i naći njegovu Laplaceovu tansformaciju.

2. Odrediti inverznu Laplaceovu transformaciju sledećih izraza:

a) $H_1(s) = \dfrac{4s^2+5}{s+3}$ b) $H_2(s) = \dfrac{2s+6}{4s^2+12s+9}$ c) $H_3(s) = \dfrac{5s+2}{s^2+s+0.89}$

3. a) Grafički odrediti konvoluciju signala

$$x_1(t) = \begin{cases} 2t & \text{za} \quad 0 < t < 10 \\ 20 & \text{za} \quad 10 < t < 60 \\ 0 & \text{za} \quad t > 60 \end{cases} \quad \text{i} \quad x_2(t) = \begin{cases} 2 & \text{za} \quad 0 < t < 30 \\ 0 & \text{za} \quad t > 30 \end{cases}$$

4. a) Izračunati koeficijente Furijeovog reda za periodični signal, periode $T = 12$

$$y(t) = \begin{cases} 2t & \text{za} \quad 0 < t < 3 \\ 12 - 2t & \text{za} \quad 3 < t < 6 \\ 0 & \text{za} \quad 6 < t < 12 \end{cases}$$ i nacrtati amplitudski i fazni spektar signala.

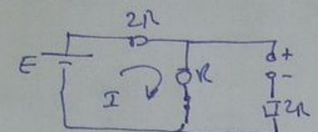
b) Nacrtati amplitudski i fazni spektar periodičnog signala $s(t) = 2 + 5\sin(2w_0t) + \cos(4w_0t + \pi/4)$

5. Odrediti odziv sistema čija je prenosna funkcija $H(s) = \dfrac{2}{s+1}$ ako se na ulaz dovede signal $x(t) = \cos(2t)$. Da li je sistem $H(s)$ stabilan?

PREDMETNI NASTAVNIK

SiS mart 2012

Z1



$$I = \frac{E}{3\Omega} = \frac{6V}{6\Omega} = \boxed{1A = i_L(0^-)}$$

$$u_c(0^-) = U_R = R \cdot I = \boxed{2V = U_c(0^-)}$$

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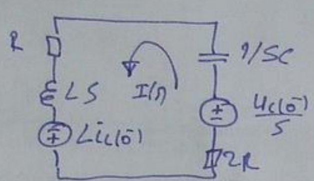
$t > 0^+$



$$I(s) = \frac{\frac{u_c(0^-)}{s} + Li_L(0^-)}{3R + Ls + \frac{1}{sC}}$$

$$I(s) = \frac{\frac{2}{s} + 3\cdot}{6 + 3s + \frac{1}{2s}} = \frac{3s+2}{3s^2+6s+\frac{1}{2}}$$

$$U_c(s) = \frac{u_c(0^-)}{s} + \frac{1}{sC}\cdot I(s) = \frac{2}{s} + \frac{1}{2s}\cdot\frac{3s+2}{3s^2+6s+\frac{1}{2}} = \frac{12s^2+24s+2+(3s+2)}{s(6s^2+12s+1)}$$

b) $$u_c(s) = \frac{12s^2+27s+4}{s(6s^2+12s+1)}$$

c) $$y(t) = 4t\cdot[u_0(t) - u_0(t-1)] + 4[u_0(t-1) - u_0(t-6)]$$

$$= 4t\,u_0(t) - u_0(t-1)[4t-4] - 4u_0(t-6) = 4t\,u_0(t) - 4(t-1)u_0(t-1) - 4u_0(t-6)$$

$$Y(s) = \frac{4}{s^2} - 4e^{-s}\cdot\frac{1}{s^2} - 4e^{-6s}\cdot\frac{1}{s}$$

2) $$H_1(s) = \frac{4s^2+5}{s+3} = 4s - 12 + \frac{41}{s+3}$$ $$\boxed{h_1(t) = 4\delta'(t) - 12\delta(t) + 41e^{-3t}\cdot u_0(t)}$$

$(4s^2+5):(s+3) = 4s - 12$
$-4s^2 \mp 12s$
$-12s+5$
$-12s-36$
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b) $$H_2(s) = \frac{2s+6}{4s^2+12s+9} \quad ; \quad s_{1,2} = \frac{-12 \pm \sqrt{144-16\cdot 9}}{8} = \frac{-12 \pm\sqrt{144-144}}{8} = -\frac{3}{2}$$

$$H_2(s) = \frac{2s+6}{4(s+1.5)^2} = \frac{r_1}{(s+1.5)^2} + \frac{r_2}{s+1.5}$$

$$r_1 = \lim_{s\to -1.5} \frac{(2s+6)}{4} = \frac{-3+6}{4} = \frac{3}{4} \qquad r_2 = \lim_{s\to-1.5} \frac{d}{ds}\frac{(2s+6)}{4} = \frac{2}{4} = \frac{1}{2}$$

$$H_2(s) = \frac{3/4}{(s+1.5)^2} + \frac{2/4}{s+1.5} = \frac{3 + 2(s+1.5)}{4(s+1.5)^2}$$

$$h_2(t) = \mathcal{L}^{-1}\{H_2(s)\} = \left[\frac{3}{4} \cdot t \cdot e^{-1.5t} + \frac{1}{2} \cdot e^{-1.5t}\right] u_0(t)$$

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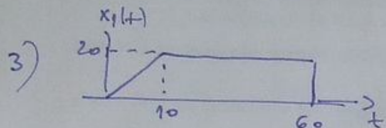
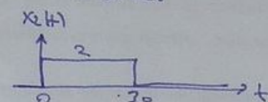
c) $$H_3(s) = \frac{5 \cdot s + 2}{s^2 + s + 0.89} = \text{konj.} = \frac{5 \cdot (s + \frac{1}{2}) - \frac{5}{2} + 2}{(s + \frac{1}{2})^2 - 0.25 + 0.89}$$

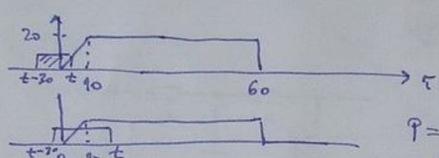
$$= \frac{5 \cdot (s + \frac{1}{2})}{(s + \frac{1}{2})^2 + 0.8^2} - \frac{1/2 \cdot \frac{0.8}{0.8}}{(s + \frac{1}{2})^2 + 0.8^2} = \frac{5 \cdot (s + 1/2)}{(s + \frac{1}{2})^2 + 0.8^2} - \frac{\frac{1}{1.6} \cdot 0.8}{(s + \frac{1}{2})^2 + 0.8^2}$$

$$h_3(t) = \mathcal{L}^{-1}\{H_3(s)\} = \left[5 \cdot e^{-0.5t} \cdot \cos(0.8t) - \underbrace{\frac{1}{1.6}}_{-0.625} \cdot e^{-0.5t} \cdot \sin(0.8t)\right] u_0(t)$$

3)







$t < 0$
$p = 0$

$0 < t < 10$

$$p = \int_0^t x_1(\tau) \cdot x_2(t-\tau)\, d\tau = \int_0^t 2\tau \cdot 2\, d\tau$$

$$= 4 \frac{\tau^2}{2} \Big|_0^t = 2\tau^2 \Big|_0^t = \boxed{2t^2}$$

za $t = 10 \Rightarrow p = 200$.

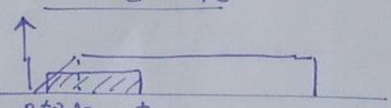
$10 < t < 30$

$$p = \int_0^t x_1(\tau) x_2(t-\tau)\, d\tau = \int_0^{10} 2\tau \cdot 2\, d\tau + \int_{10}^t 20 \cdot 2 \cdot d\tau = 200 + 40(t-10)$$

$$\boxed{= -200 + 40t}$$

za $t = 30$ $p = -200 + 1200 = 1000$

$30 < t < 40$



$$p = \int_{t-30}^{10} 2\tau \cdot 2\, d\tau + \int_{10}^t 20 \cdot 2\, d\tau = 2\tau^2 \Big|_{t-30}^{10} + 40\tau \Big|_{10}^t$$

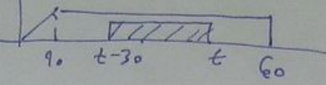
$$= 2\left[100 - (t-30)^2\right] + 40\left[t - 10\right] = 2\left[100 - (t^2 - 60t + 900)\right] + 40t - 400$$

$$= 200 - 2t^2 + 120t - 1800 + 40t - 400 = \boxed{-2t^2 + 160t - 2000}$$

za $t = 40$ $p = 1200$

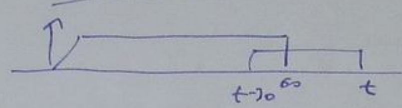
$40 < t < 60$

$$p = \int_{t-30}^{t} 20 \cdot 2 \cdot d\tau$$



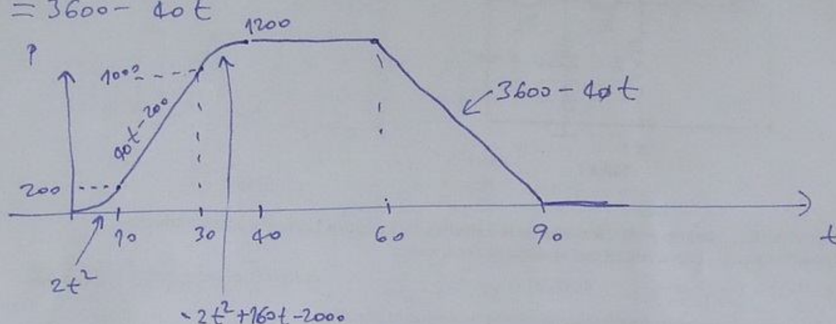
$$= 40(t-(t-30)) = 40t - 40t + 1200 = 1200$$

$60 < t < 90$

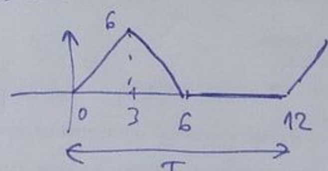


$$P = \int_{t-30}^{60} 20 \cdot 2 \, d\tau = 40\tau \Big|_{t-30}^{60} = 40(60-(t-30)) = 2400 - 40t + 1200$$

$$= 3600 - 40t$$



4) $T = 12$ $\quad y(t) =$



$$C_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{12}\left[\int_0^3 2t\,dt + \int_3^6 (12-2t)\,dt\right] = \frac{1}{12}\left[t^2\Big|_0^3 + 12t\Big|_3^6 - t^2\Big|_3^6\right]$$

$$= \frac{1}{12}\left[9 + \cancel{36} - (\cancel{36} - 9)\right] = \frac{18}{12} = \frac{3}{2}$$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt \quad ; \quad \omega_0 = 2\pi f_0 = 2\pi \cdot \frac{1}{T} = \frac{\pi}{6}$$

$$C_n = \frac{1}{12} \cdot \left[\int_0^3 2t\, e^{-jn\frac{\pi}{6}t} \cdot dt + \int_3^6 (12-2t)\, e^{-jn\frac{\pi}{6}t}\, dt\right]$$

(4)

$$= \frac{1}{6}\left[t\cdot \frac{j}{n\frac{\pi}{6}} \cdot e^{-jn\frac{\pi}{6}\cdot t} \Big|_0^3 + 6\cdot \frac{j}{n\frac{\pi}{6}} e^{-jn\frac{\pi}{6}t}\Big|_3^6 - \left[t\, \frac{j}{n\frac{\pi}{6}}\, e^{-jn\frac{\pi}{6}t}\Big|_3^6 \right.\right.$$

$$\boxed{\begin{array}{ll} t=u & dt=du \\ e^{-jn\frac{\pi}{6}t}dt = dv & v = \frac{1}{-jn\frac{\pi}{6}}\cdot e^{-jn\frac{\pi}{6}t} \end{array}}$$

$$\left.\left. - \int_3^6 \frac{1}{-jn\frac{\pi}{6}}\, e^{-jn\frac{\pi}{6}t}\, dt\right] - \int_0^3 \frac{1}{-jn\frac{\pi}{6}}\, e^{-jn\frac{\pi}{6}t}\, dt \right]$$

$$= \frac{1}{6}\left[3\cdot \frac{j}{n\pi/6}\, e^{-jn\frac{\pi}{2}} + 6\cdot \frac{j}{n\frac{\pi}{6}}\left(e^{-jn\pi} - e^{-jn\frac{\pi}{2}}\right) - \left[\left(6\cdot \frac{j}{n\frac{\pi}{6}}\, e^{-jn\pi} - 3\cdot \frac{j}{n\frac{\pi}{6}}\, e^{-jn\frac{\pi}{2}}\right)\right.\right.$$

$$\left.\left. - \frac{1}{\left(-jn\frac{\pi}{6}\right)^2}\, e^{-jn\frac{\pi}{6}t}\Big|_3^6 \right] - \frac{1}{\left(-jn\frac{\pi}{6}\right)^2}\cdot e^{-jn\frac{\pi}{6}t}\Big|_0^3 \right]$$

$$= \frac{1}{6}\left[e^{-jn\frac{\pi}{2}} \overbrace{\cancel{\left(\frac{3j}{n\cdot\frac{\pi}{6}} - \frac{6j}{n\frac{\pi}{6}} + \frac{3j}{n\frac{\pi}{6}}\right)}}^{0} + e^{-jn\pi} \overbrace{\cancel{\left(\frac{6j}{n\frac{\pi}{6}} - \frac{6j}{n\frac{\pi}{6}}\right)}}^{0} \right.$$

$$\left. + \frac{1}{-n^2\frac{\pi^2}{6^2}}\left(e^{-jn\pi} - e^{-jn\frac{\pi}{2}}\right) - \frac{1}{-n^2\frac{\pi^2}{6^2}}\left(e^{-jn\frac{\pi}{2}} - 1\right) \right]$$

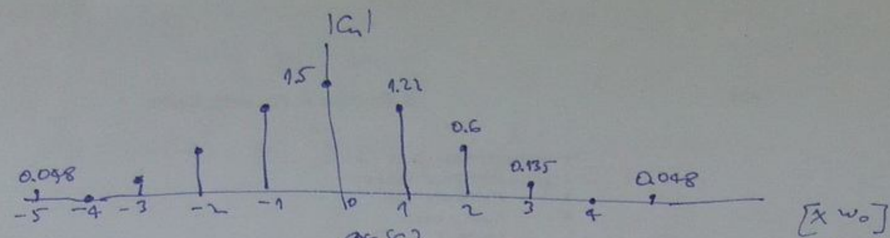
$$= \frac{1}{6}\left[\frac{-1}{\left(n\frac{\pi}{6}\right)^2}\left(e^{-jn\pi} - e^{-jn\frac{\pi}{2}}\right) + \frac{1}{\left(n\frac{\pi}{6}\right)^2}\left(e^{-jn\frac{\pi}{2}} - 1\right) \right] =$$

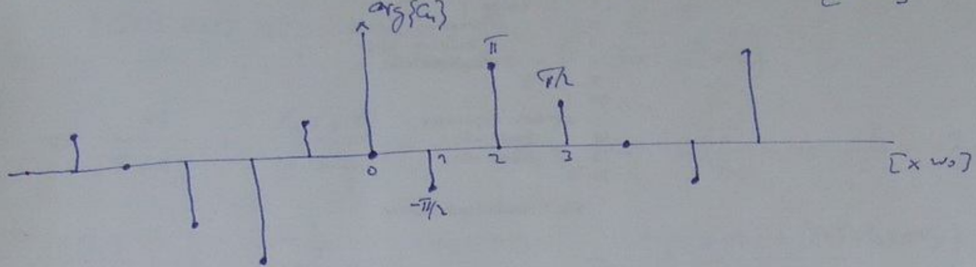
$$= \frac{1}{6}\left[\frac{2}{\left(n\frac{\pi}{6}\right)^2}\, e^{-jn\frac{\pi}{2}} - \frac{1}{\left(n\frac{\pi}{6}\right)^2}\, e^{-jn\pi} - \frac{1}{\left(n\frac{\pi}{6}\right)^2} \right] =$$

$$\frac{6}{n^2\pi^2}\left[2e^{-jn\frac{\pi}{2}} - e^{-jn\pi} - 1 \right] =$$

n	C_n	$\lvert C_n\rvert$	$\arg\{C_n\}$
1	$-1.22j$	$+1.22$	$-\pi/2$
2	-0.61	0.61	π
3	$0.135\cdot j$	0.135	$\pi/2$
4	$\varnothing$	$\varnothing$	$\varnothing$
5	$-0.048j$	0.048	$-\pi/2$
6	-0.0675	0.0675	π

n	$e^{-jn\frac{\pi}{2}}$	$e^{-jn\pi}$
1	$-j$	-1
2	-1	1
3	j	-1
4	1	1
5	$-j$	-1

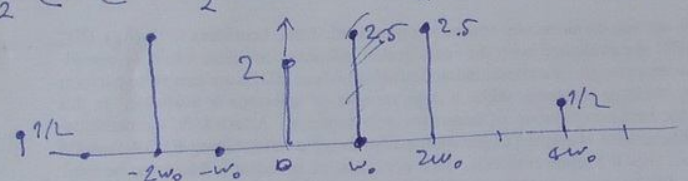


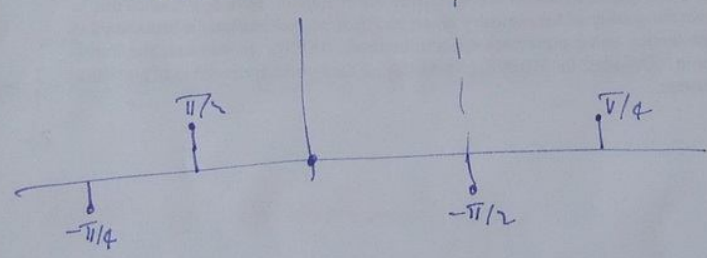


4b) $s(t) = 2 + 5\sin(2\omega_0 t) + \cos\left(4\omega_0 t + \frac{\pi}{4}\right)$

$$= 2 + 5\cdot\frac{e^{j2\omega_0 t} - e^{-j2\omega_0 t}}{2j} + \frac{e^{j4\omega_0 t + j\frac{\pi}{4}} + e^{-j4\omega_0 t - j\frac{\pi}{4}}}{2}$$

$$= 2 + \frac{5}{2}e^{-j\frac{\pi}{2}}\cdot e^{j2\omega_0 t} + \frac{5}{2}e^{j\frac{\pi}{2}}e^{-j2\omega_0 t} + \frac{1}{2}\cdot e^{j\frac{\pi}{4}}\cdot e^{j4\omega_0 t} + \frac{1}{2}e^{-j\frac{\pi}{4}}\cdot e^{-j4\omega_0 t}$$





$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s)$$

$$x(t) = \cos(2t) \Rightarrow X(s) = \frac{s}{s^2+4}$$

$$Y(s) = H(s)\cdot X(s) = \frac{s}{s^2+4}\cdot\frac{2}{s+1} = \frac{2\cdot s}{(s+1)(s^2+4)}$$

$$= \frac{r_1}{s+1} + \frac{r_2 s + r_3}{s^2+4} \qquad r_1 = \lim_{s\to -1} \frac{2s}{s^2+4} = \frac{-2}{5} = -\frac{2}{5}$$

$$\frac{2s}{(s+1)(s^2+4)} = \frac{-\frac{2}{5}}{s+1} + \frac{r_2 s + r_3}{s^2+4} = \frac{-\frac{2}{5}(s^2+4) + (s+1)(r_2 s + r_3)}{(s+1)(s^2+4)}$$

$$2s = -\frac{2}{5}s^2 - \frac{8}{5} + r_2 s^2 + s(r_2+r_3) + r_3$$

$$\begin{aligned} \emptyset &= -\frac{2}{5} + r_2 \\ 2 &= r_2 + r_3 \\ 0 &= -\frac{8}{5} + r_3 \end{aligned} \qquad \begin{aligned} r_3 &= \frac{8}{5} \\ r_2 &= \frac{2}{5} \end{aligned}$$

$$Y(s) = \frac{-\frac{2}{5}}{s+1} + \frac{\frac{2}{5}s + \frac{8}{5}}{s^2+4} = \frac{-2/5}{s+1} + \frac{2}{5}\cdot\frac{s}{s^2+4} + \frac{4}{5}\cdot\frac{2}{s^2+4}$$

$$y(t) = \left[-\frac{2}{5}\cdot e^{-t} + \frac{2}{5}\cdot\cos(2t) + \frac{4}{5}\sin(2t)\right] u_0(t)$$

Sistem je stabilan