Obrada audio signala
pismeni ispit

1. Prenosna funkcija Butterworth-ovog filtra propusnika nikih frekvencija je data izrazom

$$H(s) = \frac{1}{s^4 + 2.6131s^3 + 3.4142s^2 + 2.6131s + 1}.$$

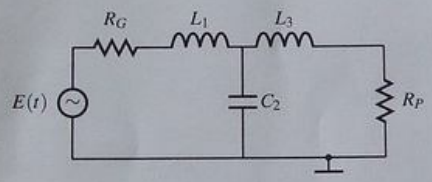
Nacrtati amplitudsku karakteristiku i realizovati filtar u obliku lestvičaste LC mreže bez gubitaka. Otpornosti generatora i potrošača iznose $R_G = R_P = 1\ \Omega$. Nacrtati šemu filtra.

2. Prenosna funkcija Butterworth-ovog filtra propusnika nikih frekvencija sa graničnom frekvencijom $f_p = 1000$ Hz, maksimalnim slabljenjem u propusnom opsegu $a_{max} = 3$ dB i minimalnim slabljenjem u nepropusnom opsegu ($f_s = 2000$Hz) $a_{min} = 30$dB je data izrazom

$$H(s) = \frac{9.9144 \cdot 10^{18}}{(s^2 + 3.893 \cdot 10^3 s + 3.9674 \cdot 10^7)(s^2 + 1.0192 \cdot 10^4 s + 3.9674 \cdot 10^7)(s + 6.2987 \cdot 10^3)}.$$

Dobijenu prenosnu karakteristiku realizovati u obliku aktivnog RC fitra sa kaskadnom spregom sekcija prvog i drugog reda i nacrtati šemu filtra.

3. Na slici je prikazana pasivna realizacija prototipske funkcije filtra propusnika niskih frekvencija. Ako je $R_G = R_p = 1$,



$L_1 = 3.03077$, $C_2 = 0.7683$, $L_3 = 3.03077$,

   (a) primenom NF→ PO transformacije realizovati filtar propusnik opsega frekvencija. Granične frekvencije propusnog opsega iznose $f_1 = 10$ kHz, $f_2 = 15$ kHz, granične frekvencije nepropusnog opsega iznose $f_3 = 7$ kHz $f_4 = 20$ kHz; $R_G = R_p = 1$ kΩ. Nacrtati šemu filtra.

4. Odrediti prenosnu funkciju Butterworth-ovog filtra propusnika niskih frekvencija čija amplitudska karakteristika zadovoljava sledeće zahteve: $\Omega_p = 1$, A_{max}=3dB, $\Omega_s = 2$, A_{min}=12dB.

5. Za zvučnik otpornosti 6Ω odrediti sve elemente i nacrtati tropojasnu skretnicu sa graničnim frekvencijama 450Hz i 4,3kHz koristeći prenosnu funkciju Butterworthovog filtra $H(s) = \frac{1}{s^2 + 1.4142s + 1}$.

U Nišu
February 29, 2012

Predmetni nastavnik
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OAS Feb. 2017.

2) $H(s) = \dfrac{1}{s^4 + 2.6131 s^3 + 3.4142 s^2 + 2.6131 s + 1}$

$T(s)T(-s) = 1 - H(s)H(-s) = 1 - \dfrac{1}{s^4+2.613s^3+3.41s^2+2.613s+1} \cdot \dfrac{1}{s^4-2.613s^3+3.41s^2-2.613s+1}$

$= \dfrac{DOLE(s)\cdot DOLE(-s) - 1}{DOLE(s)\ DOLE(-s)} = \dfrac{s^8}{DOLE(s)\ DOLE(-s)}$

$= \dfrac{s^4 \cdot (-s)^4}{DOLE(s)\cdot DOLE(-s)} \Rightarrow \boxed{T(s) = \dfrac{s^4}{s^4 + 2.6131 s^3 + 3.4142 s^2 + 2.6131 s + 1}}$

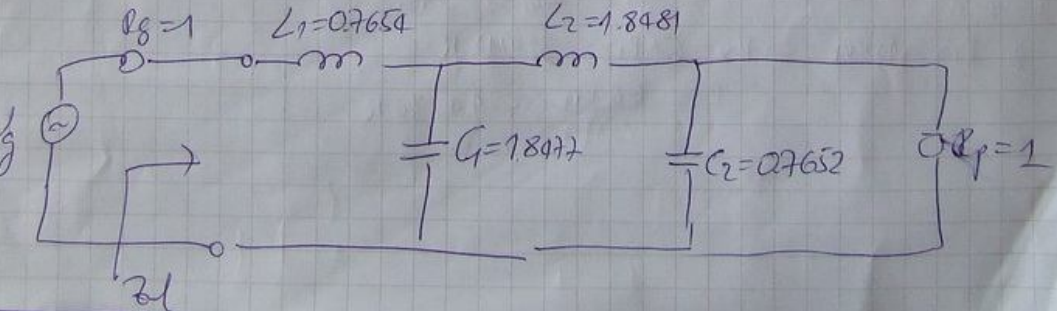
$Z_{al} = R_g \dfrac{1+T(s)}{1-T(s)} = \dfrac{1 + \frac{s^4}{DOLE(s)}}{1 - \frac{s^4}{DOLE(s)}} = \dfrac{DOLE(s) + s^4}{DOLE(s) - s^4}$

$Z_{al} = \dfrac{2s^4 + 2.6131 s^3 + 3.4142 s^2 + 2.6131 s + 1}{2.6131 s^3 + 3.4142 s^2 + 2.6131 s + 1}$

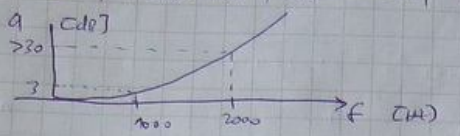
$Z_{al} = 0.7654 s + \dfrac{1.4143 s^2 + 1.8478 s + 1}{2.6131 s^3 + 3.4142 s^2 + 2.6131 s + 1}$

$= 0.7654 s + \dfrac{1}{1.8477 s + \dfrac{0.7652 s + 1}{1.4143 s^2 + 1.8478 s + 1}}$

$= 0.7654 s + \dfrac{1}{1.8477 s + \dfrac{1}{1.8489 s + \dfrac{1}{0.7652 s + 1}}}$



[2] $f_1 = 1000\,Hz$, $a_{max} = 3\,dB$, $f_2 = 2000\,Hz$, $a_{min} = 30\,dB$



$$H(s) = \frac{3.9674 \cdot 10^7}{(s^2 + 3.893 \cdot 10^3 s + 3.9674 \cdot 10^7)} \cdot \frac{3.9674 \cdot 10^7}{(s^2 + 1.0192 \cdot 10^4 s + 3.9674 \cdot 10^7)} \cdot \frac{6.2987 \cdot 10^7}{s + 6.2987 \cdot 10^7}$$

$$H_2(s) = \frac{K \cdot \frac{1}{R^2C^2}}{s^2 + s\left[(3-K)\frac{1}{RC}\right] + \frac{1}{R^2C^2}}$$

Za obe sekcije drugog reda je $\frac{1}{R^2C^2} = 3.9674 \cdot 10^7$

za $C = 100\,nF$ $\quad R = \frac{1}{C\sqrt{3.9674 \cdot 10^7}} = 1.5876\,k\Omega$

$(3-K)\frac{1}{RC} = 3.893 \cdot 10^3$ i $(3-K)\frac{1}{RC} = 1.0192 \cdot 10^4$

$K_1 = 2.3819$ $\quad K_2 = 1.3819$

$K = 1 + \frac{R_B}{R_A} \Rightarrow R_B = (K-1)R_A$

za $R_A = 10K$

$R_{B1} = 13.819\,K$ $\quad R_{B2} = 3.819\,K$

Sekcija prvog reda

$$H_1(s) = \frac{1/RC}{s + 1/RC} = \frac{6.2987 \cdot 10^7}{s + 6.2987 \cdot 10^7}$$

$\frac{1}{RC} = 6.2987 \cdot 10^7$ za $C = 100\,nF$ $\quad R = \frac{1}{C \cdot 6.2987 \cdot 10^7} = 1.587\,k\Omega$

$K_{tot} = K_1 \cdot K_2 = 3.2917$; $G_{tot} = \frac{K_{tot}}{G_{tf}} = 3.2917$

za $R_{12} = 10\,k\Omega \Rightarrow R_x = G_{tot} \cdot R_{12} = 32.917\,K$

$R_y = \frac{G_{tot} \cdot R_{12}}{G_{tot} - 1} = 14.36\,K$

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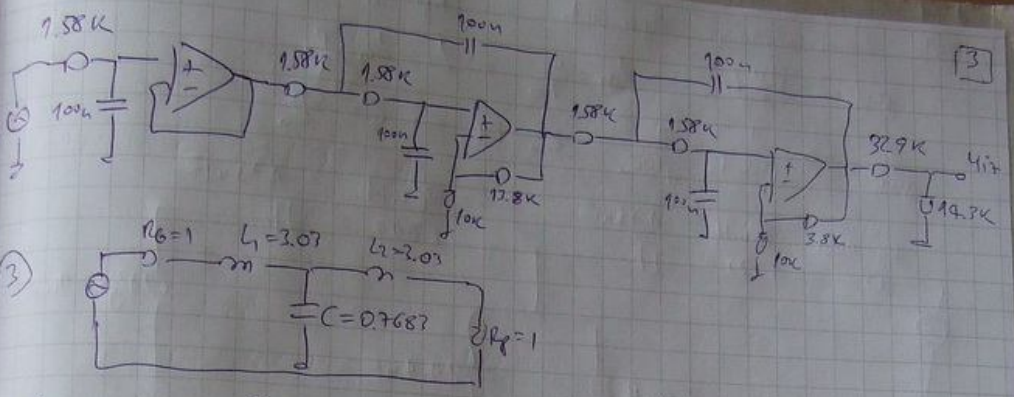


3

$f_{p1} = 10K$ $\quad f_{s1} = 7K$

$f_{p2} = 15K$ $\quad f_{s2} = 20K$

$f_{p1} \cdot f_{p2} = 150 \cdot 10^6 \neq f_{s1} \cdot f_{s2} = 140 \cdot 10^6$

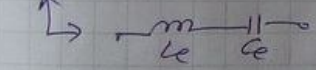
$f_{s1}^* = \frac{f_{p1} f_{p2}}{f_{s2}} = \frac{150 \cdot 10^6}{20 \cdot 10^3} = 7.5 \text{ KHz}$

$\omega_0 = 2\pi \sqrt{f_{p1} f_{p2}} = 2\pi \cdot \sqrt{150} \cdot 10^3 = 76\,953 \text{ rad/s}\ (12.2 \text{ KHz})$

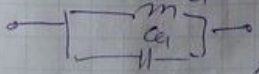
$B = 2\pi \cdot (f_{p2} - f_{p1}) = 10\,000\pi = 31\,416 \text{ rad/s}$

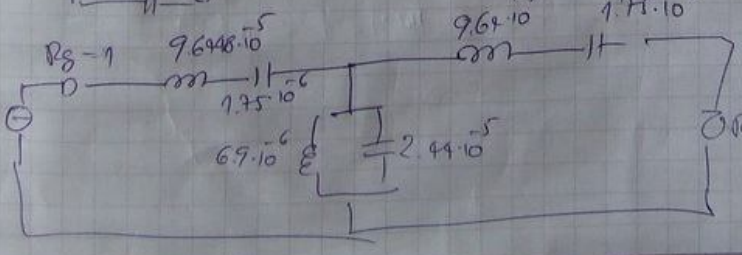
$NF \rightarrow P0 \;/\; s = \frac{s^2 + \omega_0^2}{Bs}$

$Ls \rightarrow L\frac{s^2+\omega_0^2}{Bs} = \frac{Ls}{B} + \frac{L\omega_0^2}{Bs} = Le\,s + \frac{1}{Ce\,s}$; $Le = L/B = 9.6448 \cdot 10^{-5}$; $Ce = \frac{B}{L\omega_0^2} = 1.7509 \cdot 10^{-6}$



$\frac{1}{C \cdot \frac{s^2+\omega_0^2}{Bs}} = \frac{1}{\frac{sC}{B} + \frac{C\omega_0^2}{Bs}} = \frac{1}{sCe_1 + \frac{1}{sLe_1}}$; $Ce_1 = \frac{C}{B} = 2.4456 \cdot 10^{-5}$; $Le_1 = \frac{B}{C\omega_0^2} = 6.9051 \cdot 10^{-6}$





Slika 1:

ziti preko Heaviside
laceovu transforma

b) H_2

oluciju signala
$0 < t < 10$
$10 < t < 60$
> 60

rijeovog reda za p
$0 < t < 3$
$3 < t < 6$ i nacr.
$< t < 12$

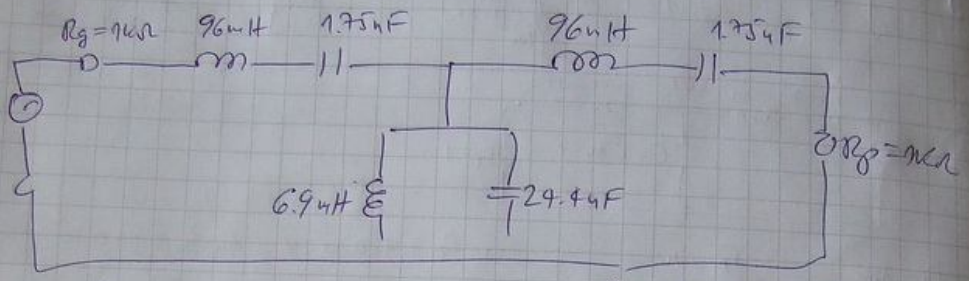
spektar periodičt.

nosna funkcija .

Prije denormalizacije otpornosti $R_p = R_g = 1k\Omega$ (4)

$j\omega L \to j\frac{\omega}{R_0} L \Rightarrow L_n = \frac{L}{R_0} \Rightarrow L = L_n \cdot R$

$\frac{1}{j\omega C} \to \frac{1}{j\omega C R_0} = \frac{1}{j\omega C_n}$ $C_n = CR_0 \Rightarrow C = \frac{C_n}{R_0}$



(4) $|H|^2 = \frac{1}{1+\varepsilon^2 \Omega^{2N}}$ za $\Omega = 1 = \Omega_p$ $|H| =$

$a_{max} = 3dB \Rightarrow A_{max} = ?$

$a = -20 \log_{10} A_{max} \Rightarrow A_{max} = 0.707$

za $a_{min} = 12dB$ $A_{min} = 0.2512$

$0.707^2 = \frac{1}{1+\varepsilon^2} \Rightarrow \varepsilon = 1$

za $\Omega = \Omega_s = 2$ $0.2512^2 \geq \frac{1}{1+\varepsilon \cdot 2^{2N}} = 0.0631 \Rightarrow 1 + 2^{2N} > 15.84$

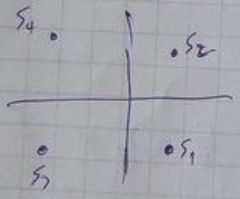
$\Rightarrow N \geq 2$

$|H|^2 = \frac{1}{1+\Omega^4}$ $s = j\Omega$, $s^2 = -\Omega^2$, $s^4 = \Omega^4$

$H(s)H(-s) = \frac{1}{1+s^4}$

$s^4 + 1 = 0 \Rightarrow s^4 = -1 = e^{-j\sqrt{\pi} + 2k\sqrt{\pi}}$

$s_{1,2,3,4} = e^{-j\frac{\sqrt{\pi}}{4} + jk\frac{\pi}{2}}$



$k=0$: $s_1 = e^{-j\frac{\pi}{4}} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} = 0.707 - j0.707$

$k=1$ $s_2 = e^{j\frac{\pi}{4}} = 0.707 + j0.707$

$k=2$: $s_3 = e^{j\frac{3\pi}{4}} = -0.707 - j0.707$

$k=3$: $s_4 = e^{-j\frac{3\pi}{4}} = -0.707 + j0.707$

$H(s)H(-s) = \frac{1}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)} \Rightarrow H(s) = \frac{K}{(s-s_3)(s-s_4)} = \frac{k}{s^2 + 1.414s + 1}$

za $s = 0$ i $|H(j)| = 1 \Rightarrow \boxed{K = 1}$

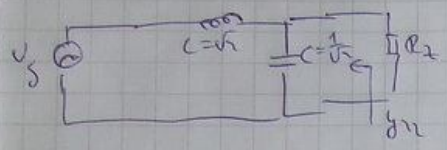
$$H(s) = \frac{\frac{1}{\sqrt{2}s}}{s^2+\sqrt{2}s+1} = \frac{\frac{1}{\sqrt{2}s}}{1+\frac{s^2+1}{\sqrt{2}s}}$$

$y_{21} = -\frac{1}{\sqrt{2}s}$

ima nule u $s \to \infty$

5

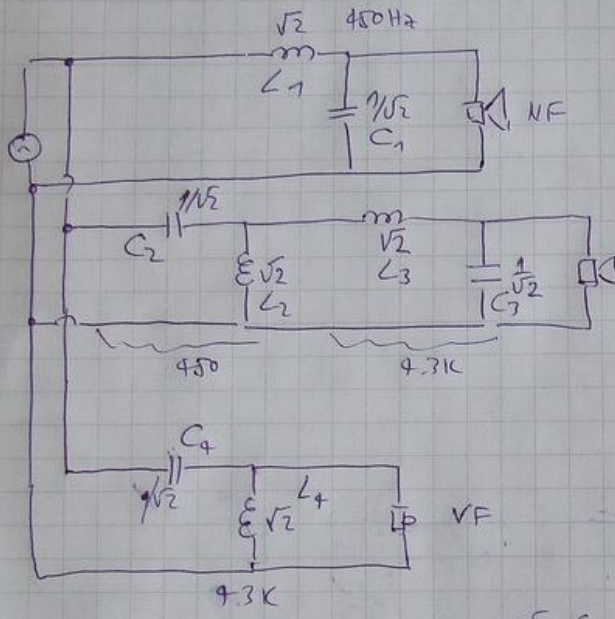
$$y_{21} = \frac{s^2+1}{\sqrt{2}s} = \frac{1}{\sqrt{2}}\cdot s + \frac{1}{\sqrt{2}s}$$



NF → VF $\quad s \to \frac{1}{s}$

$sL \to \frac{1}{s}L = \frac{1}{sC_e}$; $C_e = 1/L$

$\frac{1}{sC} \to \frac{s}{C} = sL_e$; $L_e = 1/C$



$$L = \frac{L_1 \cdot R_0}{\omega_0}, \quad R_0 = 6\Omega$$

$$C = \frac{C_1}{\omega_0 R_0}$$

Za $\omega_0 = (450\ Hz) = 2827\ \frac{rad}{s}$

$L_1 = \frac{\sqrt{2}\cdot 6}{\omega_0} = 3\,mH$

$C_1 = 41.7\,\mu F$

$C_2 = C_1 \quad L_2 = L_1$

Za $\omega_0 = (4300\ Hz) = 27018\ \frac{rad}{s}$

$L_3 = 314\,\mu H = L_4$

$C_3 = 9.3\,\mu F = C_4$